Solution to Test 1(A)

MAT1320E, Fall 2015
Total = 20 marks

Part I. Short Answer Questions: (14 marks)

Only the final answer is to be marked.

1. (3 marks) Assume some values of a one-to-one function $y = f(x)$ are given in the following table

x	1	2	3	4	5
$f(x)$	4	5	1	2	3
$f'(x)$	2	5	1	3	4

Let $g = f \circ f$, and $h = f^{-1}$.

(a) $g(2) =$ ______________________ .

(b) $h(2) =$ ______________________ .

Solution. (a) $g(2) = f(f(2)) = f(5) = 3$.

(b) Since $f(4) = 2, f^{-1}(2) = 4$.

2. (2 marks) The limit $\lim_{x \to -1} \dfrac{2x^2 - x - 3}{x^2 - 1} =$ ____________________ .

Solution:

$$\lim_{x \to -1} \frac{2x^2 - x - 3}{x^2 - 1} = \lim_{x \to -1} \frac{(x+1)(2x-3)}{(x-1)(x+1)} = \lim_{x \to -1} \frac{2x-3}{x-1} = \frac{5}{2}$$

3. (2 marks) If a tangent line of the graph of function $y = x^2$ is parallel to the line $2x + y = 1$. The equation of the tangent line is

$y =$ ______________________________ .

Solution. The slope of the given line is -2. $y' = 2x = -2$. Then $x = -1$. $y = 1$. The equation of the tangent line is $y = -2(x + 1) + 1$, or $y = -2x - 1$.

4. (2 marks) Let $f(x) = \frac{e^x}{x^2}$. Then $f'(x) =$__________________ .

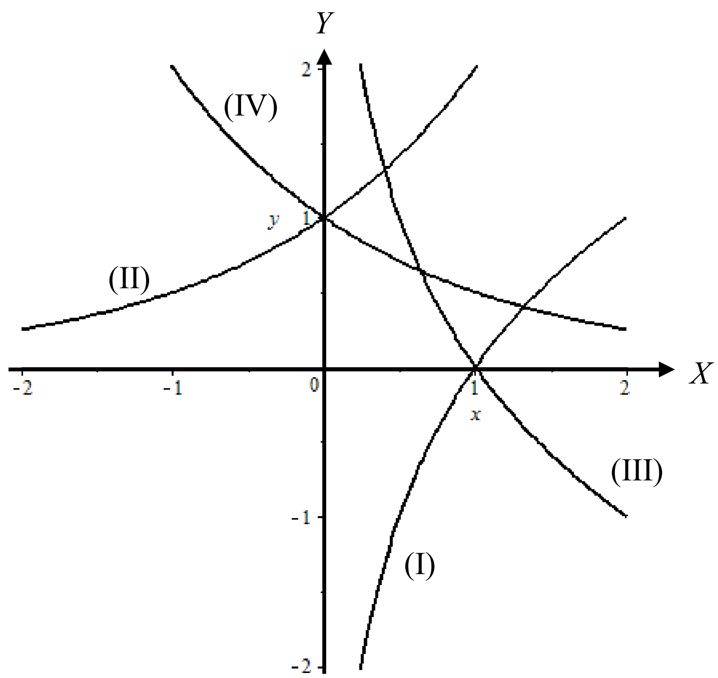
Solution. $f'(x) = \dfrac{e^x x^2 - 2xe^x}{x^2}$

5. (2 marks) Solve equation $\log_2(x + 3) - \log_2(2x + 5) = -2$. Then $x =$ _______________ .

Solution. $\log_2 \dfrac{x+3}{5x+4} = -2$, $\dfrac{x+3}{5x+4} = \dfrac{1}{4}$, $4x + 12 = 5x + 4$, $x = 8$.

6. (2 marks) Match the following graphs with functions

(a) $y = 2^x$, (b) $y = \left(\frac{1}{2}\right)^x$, (c) $y = \log_2 x$, (d) $y = \log_{(1/2)} x$:



Answer. (I) -> (c), (II) -> (a), (III) -> (d), (IV) -> (b).

7. (1 mark) The range of function $y = \arctan x$ is ____________________________ .

Solution: The range is $(-\pi/2, \pi/2)$. We could also find arctan of other intervals, we just have to make sure that the interval we choose is one-to-one.

Part II. Detailed Answer Question

1. (3 marks) **Use the definition of the derivative** to find the derivative of the function $y = \dfrac{1}{\sqrt{x}}$.

Solution. $y' = \lim_{h\to 0}\dfrac{1}{h}\left(\dfrac{1}{\sqrt{x+h}} - \dfrac{1}{\sqrt{x}}\right) = \lim_{h\to 0}\dfrac{\sqrt{x}-\sqrt{x+h}}{h\sqrt{x(x+h)}} = \lim_{h\to 0}\dfrac{(\sqrt{x}-\sqrt{x+h})(\sqrt{x}+\sqrt{x+h})}{h\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})}$

$= -\lim_{h\to 0}\dfrac{h}{h\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})} = -\lim_{h\to 0}\dfrac{1}{\sqrt{x(x+h)}(\sqrt{x}+\sqrt{x+h})} = -\dfrac{1}{2x\sqrt{x}}$.

2. (3 marks) Find all vertical and horizontal asymptotes of the graph of the function $y = \dfrac{2x^2+1}{x^2-x}$.

Solution. Let $x^2 - x = 0$, $x = 0, 1$. This function has vertical asymptotes $x = 0$, and $x = 1$.

$\lim_{x\to\infty}\dfrac{2x^2+1}{x^2-x} = \lim_{x\to-\infty}\dfrac{2x^2+1}{x^2-x} = 2$. This function has horizontal asymptote $y = 2$.